Comparison of different solution techniques of the 2-D Steady State Inverse Heat Conduction Problem

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Introduction

The solution of the 2-D Steady state Inverse Heat Conduction Problem (SIHCP), by using the surface temperature distribution as input data, enables to estimate an important parameter in heat exchangers design, that is the heat flux density on a given surface and then the corresponding local heat transfer coefficient. Although SIHCP can be regarded as a special case of the Unsteady state Inverse Heat Conduction Problem (UIHCP), some caution is needed in the application of standard inverse solution strategies to this particular problem. The peculiarities of this approach are related to the destructive effect of noise which is amplified in the steady case by the necessity of estimating the wanted information from the signal Laplacian and not from the signal first temporal derivative, as it often happens in the classical formulation of the UIHCP.

Among the techniques available in literature for the solution of inverse heat transfer problems, the “forced matching technique”, the “Wiener filtering technique” and the “Conjugate Gradient Method with adjoint problem formulation” appear as the most suitable for this particular application.

In the “forced matching technique”, the temperature distribution obtained by numerically solving the corresponding direct problem, is forced to match the experimental noisy data, [1,2]. The “Wiener filtering technique”, consisting in two consecutive applications of the Wiener filter, removes from the raw temperature data the unwanted experimental noise by makeing the direct calculation of the signal’s Laplacian feasible, [3,4]. Last, the “Conjugate Gradient Method with adjoint problem formulation” (CGM) handles the ill-posed nature of the problem by reformulating the problem as a well-posed problem by minimizing the squared difference between measured and estimated temperature discrete data [5].

Analysis, Results and Conclusions

In the present work a simulated noisy signal, representing the experimental input data of a SIHCP with convective thermal boundary conditions, is considered in order to compare the above referenced solution techniques. In particular the temperature distribution occurring on a rectangular thin plate, exposed on one side to a forced turbulent flow and thermally insulated on the other is evaluated. On the external frame of the plate a first type thermal boundary condition has been imposed.

The exact solution of the problem has been obtained by numerically solving the direct problem under a standard finite difference approach. The free-stream wall temperature difference, in this condition, ranges from 2K to 10K and a significant spatial variation of the surface heat flux occurs. The signal has been mapped on a discrete domain with 600×400 equally spaced nodes in order to handle the problem formulation in which the number of unknowns is high, as it frequently occurs in applications based on highly spatially resolved temperature maps acquired by means thermographic systems. A random additive noise with a standard deviation of 0.1 K has been imposed. The distributions of the noisy test signal is reported in Fig.1.

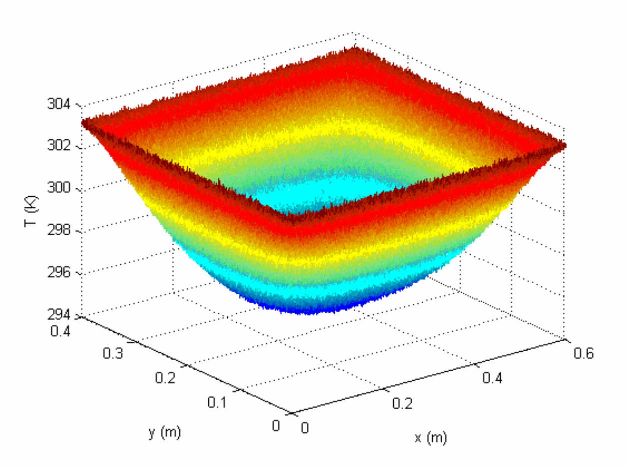

Figure 1. Noisy test signal.

The comparison, in terms both of surface heat flux and of convective heat transfer coefficient distribution along the domain centerline, between the exact solution and the restored discrete function obtained though the three above describe estimation methodologies is reported in Figs 2-3-4. Figure 5 shows the representative reconstructed temperature distribution along the domain centerline, obtained by applying the CGM. An analogous filtering effect on the raw data is associated to the other solution techniques.

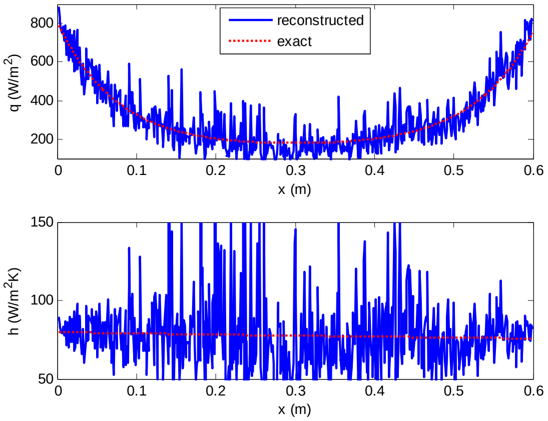

Figure 2. Forced matching technique.

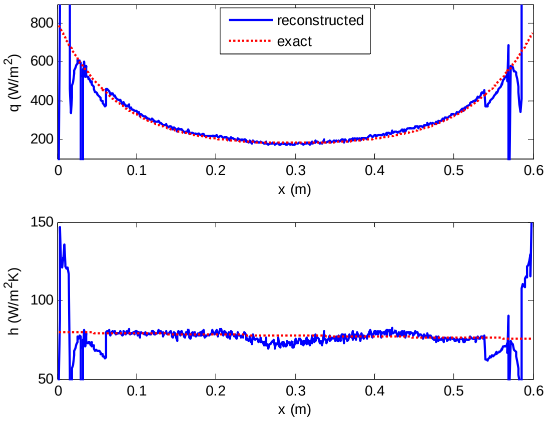

Figure 3. Wiener filtering technique.

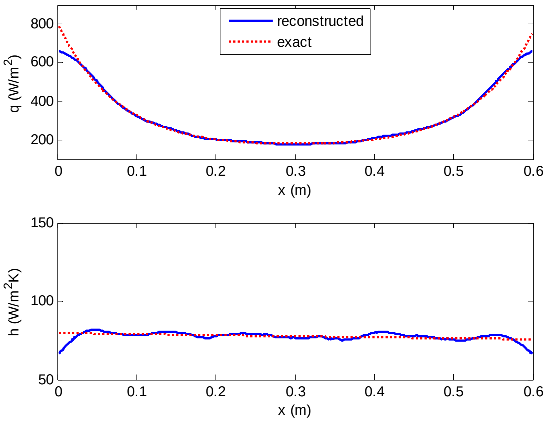

Figure 4. CGM with adjoint problem formulation.

The CGM, if compared to the other techniques, shows, with regards to this problem, a better performance. The "Wiener filter" techniques shows a good performance even if it confirms its well-known limit close to the border of the domain [4]. This problem affects also the CGM but it can be partially bypassed by choosing suitable data for the starting value of the unknown in the formulation of the solution algorithm.

The "forced matching technique" shows a poor effectiveness in terms of local estimation capability, although the average values are restored with sufficient accuracy.

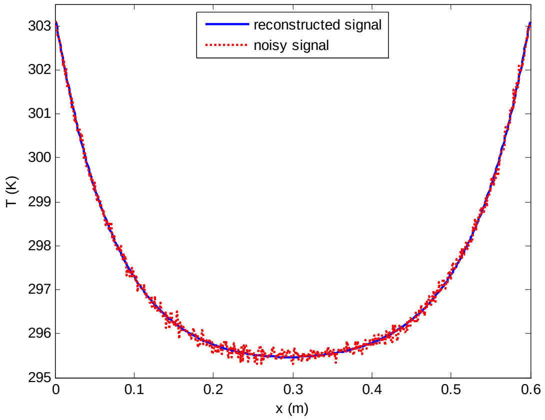

Figure 5. Noisy and reconstructed signal (CGM).

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